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Higgs and neutrino sector, EDM and ϵ_K in a spontaneously CP and R-parity breaking supersymmetric model

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Abstract. We construct an extension of the supersymmetric standard model where both CP symmetry and R-parity are spontaneously broken. We study the electroweak symmetry breaking sector of the model and find minima consistent with the experimental bounds on Higgs boson masses. Neutrino masses and mixing angles are generated through both seesaw and bilinear R-parity violation. We show that the hierarchical mass pattern is obtained, and mixings are consistent with measured values. Due to the spontaneous CP and R-parity violation, the neutrino sector is CP violating, and we calculate the corresponding phase. We further restrict the parameter space to agree with the limits on the electric dipole moment of the neutron. Finally, we study the CP violation parameter ϵ_K in the kaon system and show that we obtain results consistent with the experimental value.

1 Introduction

While the standard model of electroweak interactions (SM) has achieved a great deal of success, there still exist fundamental questions for which it does not provide answers.

One of these challenging questions is charge-parity (CP) violation. Though at present the Cabibbo–Kobavashi–Maskawa (CKM) mechanism is in agreement with existing experimental data, it cannot accommodate the baryon asymmetry of the universe needed in big bang cosmology [1]. There is thus need for physics coming from beyond the SM scenarios. There are two basic possibilities to break CP: explicit at the Lagrangian level, or spontaneous in the vacuum [2]. The SM represents a case in which CP is broken explicitly [3], through the introduction in the Lagrangian of complex Yukawa couplings which lead to CP violation in the charged-current weak interactions. Extensions of the SM which introduce new CP violating phases often lead to phenomenological difficulties. For instance, a general two Higgs doublet model with acceptable flavor changing interactions predicts a too large value for ϵ_K [4]. In supersymmetric (SUSY) extensions of the SM, one has additional sources of explicit CP violation arising from soft supersymmetry breaking terms in the Lagrangian. While in supersymmetric models a large number of new phases emerge [5], in a general minimal supersymmetric standard model (MSSM) these phases are strongly constrained by electric dipole moments (EDMs) [6–8]. Thus, in general, supersymmetric models share the problem of the origin of CP violation with the standard model and generate too large supersymmetric contributions to the dipole moments (known as the SUSY CP problem) [9–11].

An alternative scheme which could explain the source of CP violation is achieved through spontaneous symmetry breaking (SCPV) [2]. In this scenario the Lagrangian is invariant under the CP symmetry, but the ground state is asymmetric and the only sources of CP violation are the vacuum phases. Another motivation for SCPV arises from the strong CP problem. In spontaneous CP breaking $\overline{\Theta}$ vanishes at tree level and is calculable at higher orders [12–14]. Additional justification for studying SCPV comes from string theories, where CP exists as a good symmetry, but could be broken spontaneously [15]. Models with spontaneous CP violation require an extension of the minimal Higgs structure of the SM [16-23]. In general, more than one neutral Higgs particle will participate in flavor changing interactions. However, these interactions are severely restricted by the smallness of the $K_L - K_S$ mass difference. One must have a mechanism to suppress such contributions either by extending neutral flavor conservation to the Higgs sector (requiring vanishing of flavor changing couplings) or requiring the neutral Higgs bosons to be heavy [24].

Another problem of the standard model, which persists in the MSSM, is that the neutrino masses vanish. Yet the neutrino experiments have provided strong evidence for small nonvanishing neutrino masses [25]. Perhaps the

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most popular mechanism to explain neutrino masses is the seesaw mechanism [27–34], which can generate small masses for neutrinos by allowing Majorana masses through the introduction of heavy right-handed neutrinos. Another popular way to explain the neutrino masses is through a small violation of *R*-parity [35], $R_{\rm p} = (-1)^{3B-L+2s}$, where B = baryon number, L = lepton number, and s =spin of the particle [36]. Extensive phenomenological studies of *R*-parity violating effects exist; for a recent review see [26].

While there is no fundamental reason for the existence of R-parity in the MSSM, it is put in by hand in order to protect the proton from decaying. However models beyond the MSSM exist, which allow for R-parity violation in the lepton sector only. Such is the case if R-parity is broken spontaneously [37]. If only lepton number (or baryon number) is violated, the proton does not decay. In, e.g., [38], phenomenological implications of a model with spontaneous R-parity violation in the quark and leptonic sectors were explored.

Both neutrino masses and CP violation could be explained if CP and *R*-parity were symmetries of the Lagrangian, but spontaneously broken by the vacuum. In this paper we consider a model where both of these violations are intertwined and both are spontaneous. CP is broken spontaneously through complex vacuum expectation values (VEVs) of the neutral scalar bosons, while *R*-parity violation and the seesaw mechanism will be allowed to provide the required masses and mixing parameters for neutrinos. We show that while it is nontrivial to satisfy conditions for both symmetries to break down at the same time, there are regions in the parameter space where suitable Higgs masses, as well as measured neutrino mass differences and mixing angles are obtained. In such a model, CP is violated in the neutrino sector.

A model which included both neutrino mass generation mechanisms – namely seesaw and R-parity violation – with spontaneous R-parity violation was realized in [39], where the spontaneous R-parity violation was introduced via a term proportional to NLH_2 . This term represents the familiar bilinear R-parity violating term mixing lepton and Higgs superfields, LH_2 , when the right-handed sneutrino field, \tilde{N} , develops a vacuum expectation value. This term, however, also breaks lepton number spontaneously, and thus introduces a superfluous massless Goldstone boson into the scalar spectrum. The problem can be solved by adding a singlet S to the theory, through a term N^2S , which explicitly breaks lepton number (but not R-parity), if S is not assigned lepton number -2.

Attempts to violate CP spontaneously, by complex VEVs of the neutral scalars, exist [2], but fulfilling the experimental constraints has proven difficult. More than one Higgs doublet is needed, see, e.g., [24]. Spontaneous breaking of CP is not possible at tree level in the MSSM with two Higgs doublets, nor it is allowed in a model with four doublets [40]. Studies of minimal CP violations in the MSSM have shown that if no other symmetries are imposed, at least two extra singlet fields are required [41]. Instead of

adding doublets, or two extra singlets, one can study extended models, like the NMSSM model of [42-44], where the so called μ -problem has been avoided by adding one singlet and requiring \mathbf{Z}_3 symmetry. At tree-level one cannot get spontaneous CP violation in this model either, and consequently radiative corrections must be evoked [45]. In that case a very light Higgs boson emerges [46] as it also happens in the MSSM, if spontaneous CP violation is induced via radiative corrections [47, 48]. The consequences on the Higgs boson mass were also explored in [49]. Another possibility studied is to discard the \mathbf{Z}_3 symmetry completely. On one hand, this way one loses the solution to the μ -problem, but on the other hand, it is possible to achieve SCPV [50-53] and also solve the problem of domain walls, which are created during the EW phase transition as the \mathbf{Z}_3 symmetry is broken spontaneously.

An interesting model for spontaneous CP violation was presented in [54, 55], where the \mathbf{Z}_3 symmetry is replaced by *R*-symmetries on the whole superpotential, including non-renormalizable terms [56]. The method generates a \mathbf{Z}_3 breaking tadpole term for the singlet field *S* in the soft SUSY breaking part of the Lagrangian. This tadpole term allows for spontaneous CP violation to occur at tree-level [41, 54, 55]. The tadpole is assumed to originate from non-renormalizable interactions, which do not spoil quantum stability. We adopt this approach here.

Several models in the literature have explored the consequences of breaking CP spontaneously. Some have studied the effects on leptonic observables, such as neutrino masses in the presence of right-handed neutrinos [57, 58] and leptogenesis [59]; as well as the effects on the electric dipole moments [60]. The consequences of having complex phases in the VEVs of the Higgs bosons have been analyzed in the kaon system [61] and B-meson system [62, 63]. The consequence of allowing spontaneous, rather than explicit, CP breaking, is that the CKM matrix obtained is real. While several models mentioned above can generate CP violation in the kaon system that is consistent with the experimental data, a recent study argues that the CKM matrix is likely complex [64]; consequently a "hybrid" model was constructed in which more than one source of CP violation is present, allowing both a complex CKM matrix and non-trivial CP phases in the Higgs potential [65, 66]. Here we do not study the B-sector in detail and we expect that modifications in quark sector are needed, e.g., along the lines discussed above, to fulfill all experimental results. However, these changes do not qualitatively change the results obtained in this work, as we discuss later.

Our paper is organized as follows. We give the Lagrangian and describe the model we used in Sect. 2. We explore Higgs boson masses and impose the condition that the masses satisfy experimental bounds in Sect. 3. We show that using both seesaw and R-parity violation, correct neutrino masses and mixings are obtained, in Sect. 4. We also calculate the Jarlskog invariant of the neutrino sector. We explore the consequences of CP violation in the model and show that we can obtain a region of the parameter space compatible with the bounds on the electric dipole moments and obtain the observed ϵ_K in the kaon system in Sect. 5. We conclude in Sect. 6.

2 The model

Our model is based on the superpotential

$$W = \varepsilon_{\alpha\beta} \left(h_U^{ij} Q_i^{\alpha} H_2^{\beta} U_j + h_D^{ij} H_1^{\alpha} Q_i^{\beta} D_j + h_E^{ij} H_1^{\alpha} L_i^{\beta} E_j + h_N^{ij} L_i^{\alpha} H_2^{\beta} N_j + \lambda_H H_1^{\alpha} H_2^{\beta} S \right) + \frac{\lambda_S}{3!} S^3 + \frac{\lambda_{N_i}}{2} N_i^2 S , \qquad (1)$$

where H_1 and H_2 denote the Higgs doublet superfields, L_i and Q_i the left-handed lepton and quark doublet superfields, respectively, and E_i and U_i , D_i the lepton and quark singlet superfields. Right-handed neutrino superfields are denoted by N_i , i = 1, 2, 3, and S is the gauge singlet superfield. The SU(2) contraction is defined as $\varepsilon_{12} = -\varepsilon_{21} = 1$. Assuming baryon number conservation and \mathbb{Z}_3 symmetry, the terms in the superpotential are the only renormalizable ones that respect CP and R-parity, in addition to the gauge symmetry. All the parameters in the Lagrangian are real.

The soft SUSY breaking terms in this model are the mass terms for scalars and gauginos, trilinear A-terms, and the additional S-tadpole,

$$-V_{\text{soft}} = M_Q^{ij2} \tilde{Q}_i^* \tilde{Q}_j + M_U^{ij2} \tilde{U}_i^* \tilde{U}_j + M_D^{ij2} \tilde{D}_i^* \tilde{D}_j + M_L^{ij2} \tilde{L}_i^* \tilde{L}_j + M_E^{ij2} \tilde{E}_i^* \tilde{E}_j + M_N^{ij2} \tilde{N}_i^* \tilde{N}_j + M_S^2 S^* S + m_{H_1}^2 H_1^* H_1 + m_{H_2}^2 H_2^* H_2 - \frac{1}{2} (M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{h.c.}) + \left[\varepsilon_{ab} (A_U^{ij} \tilde{Q}_i^a \tilde{U}_j H_2^b + A_D^{ij} \tilde{Q}_i^a \tilde{D}_j H_1^b + A_E^{ij} \tilde{L}_i^a \tilde{E}_j H_1^b + A_N^{ij} \tilde{L}_i^a \tilde{N}_j H_2^b + A_H H_1^a H_2^b S) \right] + \sum_i \frac{A_{N_i}}{2} S \tilde{N}_i^2 + \frac{A_S}{3!} S^3 + \xi^3 S + \text{h.c.} \right].$$
(2)

Here i, j run over the family indices. In the tadpole term, $\xi^3 S$, the parameter ξ , which originates from nonrenormalizable terms [54, 55], has been taken to be a free parameter, of the order of the soft supersymmetry breaking terms. We impose a flavor diagonal texture on $h_{U,D,E}^{ij}$ and the corresponding A-terms. The full tree-level scalar potential is $V_s = V_{\text{soft}} + V_F + V_D$, where V_F and V_D are the usual F and D terms. All together, the model contains in the superpotential 51 additional parameters compared to the general MSSM. If we had instead the MSSM with additional explicit R-parity violating terms, there would be 48 new couplings [26]. If in addition CP would be broken, a large number of phases would appear in the soft masses and couplings. Compared to these numbers of parameters, our model with spontaneous CP and R-parity violation is economical.

The minimization of the scalar potential with respect to the moduli of the scalar fields ϕ_i and the corresponding phases θ_i yield constraints later used in finding the scalar mass matrix,

$$\frac{\partial V_s}{\partial \phi_i}\Big|_{\phi=\langle \phi \rangle} = 0, \quad \frac{\partial V_s}{\partial \theta_i}\Big|_{\phi=\langle \phi \rangle} = 0.$$
(3)

Without spontaneous CP violation, the VEVs are real and the minimization equations with respect to the phases are always satisfied. The minimization equations for the charged scalars can be trivially solved by setting all charged scalar VEVs to zero. As long as the tree-level masses of these fields remain positive and the corresponding soft A-terms remain small enough, this is also the global minimum of the potential with respect to these fields [39]. The complex VEVs remain free parameters and we denote (the phase of H_1 can always be rotated away):

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\delta_2} \end{pmatrix}, \quad \langle S \rangle = \sigma_S e^{i\theta_S},$$
$$\langle \tilde{L}_i \rangle = \begin{pmatrix} \sigma_{L_i} e^{i\theta_{L_i}} \\ 0 \end{pmatrix}, \quad \langle \tilde{N}_i \rangle = \sigma_{R_i} e^{i\theta_{R_i}}.$$
(4)

Note that since $R_{\rm p}$ is violated, the W mass is $m_W^2 = \frac{1}{2}g_2^2v^2$ where $v^2 \equiv v_1^2 + v_2^2 + \sigma_{L_i}^2 \approx (174\,{\rm GeV})^2$. Minimizing with respect to the neutral scalars, we get

Minimizing with respect to the neutral scalars, we get at the tree-level:

$$\begin{split} \frac{\partial V}{\partial v_1} &= 2v_1 \left(m_{H_1}^2 + \lambda_H^2 \left(v_2^2 + \sigma_S^2 \right) + \frac{g_1^2 + g_2^2}{4} \left(v_1^2 - v_2^2 + \sigma_{L_i}^2 \right) \right. \\ &+ \tan \beta \left(\frac{1}{2} \lambda_H \lambda_S \sigma_S^2 \cos(\delta_2 - 2\theta_S) \right. \\ &+ \frac{1}{2} \lambda_H \lambda_{N_i} \sigma_{R_i}^2 \cos(\delta_2 + 2\theta_{R_i}) \\ &+ A_H \sigma_S \cos(\delta_2 + \theta_S) \right) \right) \\ &+ 2\lambda_H h_N^{ij} \sigma_S \sigma_{L_i} \sigma_{R_j} \cos(\theta_S - \theta_{L_i} + \theta_{R_j}) \,, \\ \frac{\partial V}{\partial v_2} &= 2v_2 \left(m_{H_2}^2 + \lambda_H^2 \left(v_1^2 + \sigma_S^2 \right) - \frac{g_1^2 + g_2^2}{4} \left(v_1^2 - v_2^2 + \sigma_{L_i}^2 \right) \right. \\ &+ h_N^{ij} h_N^{ik} (\sigma_{R_j} \sigma_{R_k} \cos(\theta_{R_j} - \theta_{R_k}) \\ &+ \sigma_{L_j} \sigma_{L_k} \cos(\theta_{L_j} - \theta_{L_k}) \right) \\ &+ \cot \beta \left(\frac{1}{2} \lambda_H \lambda_S \sigma_S^2 \cos(\delta_2 - 2\theta_S) \right. \\ &+ \frac{1}{2} \lambda_H \lambda_{N_i} \sigma_{R_i}^2 \cos(\delta_2 + 2\theta_{R_i}) \\ &+ A_H \sigma_S \cos(\delta_2 + \theta_S) \right) \right) \\ &+ 2A_N^{ij} \sigma_{L_i} \sigma_{R_j} \cos(\delta_2 - \theta_S + \theta_{L_i} + \theta_{R_j}) \,, \\ \frac{\partial V}{\partial \sigma_S} &= 2\sigma_S \left(m_S^2 + \lambda_H^2 \left(v_1^2 + v_2^2 \right) \right. \\ &+ \lambda_S \lambda_H v_1 v_2 \cos(\delta_2 - 2\theta_S) + \lambda_{N_i}^2 \sigma_{R_i}^2 \\ &+ \frac{1}{2} A_S \sigma_S \cos(3\theta_S) + \frac{1}{2} \lambda_S^2 \sigma_S^2 \right) \end{split}$$

$$\begin{split} &+ \frac{1}{2} \lambda_{S} \lambda_{N_{i}} \sigma_{R_{i}}^{2} \cos(2\theta_{S} + 2\theta_{R_{i}}) \bigg) \\ &- 2\xi^{3} \cos(\theta_{S}) + 2A_{H}v_{1}v_{2} \cos(\delta_{2} + \theta_{S}) \\ &+ A_{N_{i}}\sigma_{R_{i}} \cos(\theta_{S} - 2\theta_{R_{i}}) \\ &+ 2h_{N}^{ij}\sigma_{L_{i}}\sigma_{R_{j}} (\lambda_{H}v_{1} \cos(\theta_{S} - \theta_{L_{i}} + \theta_{R_{j}})) \\ &+ \lambda_{N_{j}}v_{2} \cos(\delta_{2} - \theta_{S} - \theta_{L_{i}} + \theta_{R_{j}})) , \end{split}$$

$$\begin{aligned} \frac{\partial V}{\partial \sigma_{L_{i}}} &= 2\sigma_{L_{i}} \bigg(M_{L_{i}}^{2} + \frac{g_{1}^{2} + g_{2}^{2}}{4} (v_{1}^{2} - v_{2}^{2} + \sigma_{L_{j}}^{2}) \bigg) \\ &+ 2A_{N}^{ij}v_{2}\sigma_{R_{j}} \cos(\delta_{2} + \theta_{L_{i}} - \theta_{R_{j}}) \\ &+ 2\lambda_{H}h_{N}^{ij}v_{1}\sigma_{S}\sigma_{R_{j}} \cos(\theta_{S} - \theta_{L_{i}} + \theta_{R_{j}}) \\ &+ 2h_{N}^{ij}h_{N}^{k}v_{2}^{2}\sigma_{L_{j}} \cos(\theta_{L_{i}} - \theta_{L_{j}}) \\ &+ 2h_{N}^{ij}h_{N}^{k}v_{2}^{2}\sigma_{L_{j}} \cos(\theta_{L_{i}} - \theta_{L_{j}} + \theta_{R_{i}} - \theta_{R_{j}}) \\ &+ 2h_{N}^{ij}h_{N}^{k}v_{2}\sigma_{\sigma}\sigma_{R_{j}} \cos(\theta_{S} - 2\theta_{R_{i}}) \\ &+ 2h_{N}^{ij}h_{N}^{k}\sigma_{R_{j}}\sigma_{L_{k}}\sigma_{R_{i}} \cos(\theta_{L_{i}} - \theta_{L_{k}} + \theta_{R_{i}} - \theta_{R_{j}}) \\ &+ \lambda_{N_{i}}\lambda_{N_{j}}\sigma_{R_{j}}^{2} \cos(2\theta_{R_{i}} - \theta_{R_{j}}) \\ &+ \lambda_{N_{i}}\lambda_{N_{j}}\sigma_{R_{j}}^{2} \cos(2\theta_{R_{i}} - \theta_{R_{j}}) \\ &+ \lambda_{N_{i}}\lambda_{N_{j}}\sigma_{R_{j}}^{2} \cos(2\theta_{R_{i}} - \theta_{R_{k}}) \\ &+ 2A_{N}^{ij}h_{N}^{k}v_{2}\sigma_{R_{k}} \cos(\theta_{R_{i}} - \theta_{R_{k}}) \\ &+ 2A_{N}^{ij}h_{N}^{k}v_{2}\sigma_{R_{k}} \cos(\theta_{R_{i}} - \theta_{R_{k}} + \theta_{L_{j}}) \\ &+ 2h_{N}h_{N}^{ij}h_{N}v_{2}\sigma_{R_{j}} \cos(\delta_{2} - \theta_{S} + \theta_{R_{i}} + \theta_{L_{j}}) \\ &+ 2h_{N}^{ij}h_{N}^{k}v_{L_{j}}\sigma_{R_{k}}\sigma_{L_{l}} \cos(\theta_{R_{i}} - \theta_{R_{k}} + \theta_{L_{l}} - \theta_{L_{j}}) , \\ \frac{\partial V}{\partial \delta_{2}} &= \lambda_{H}v_{1}v_{2} \Big(-\lambda_{S}\sigma_{S}^{2} \sin(\delta_{2} - 2\theta_{S}) \\ &- \lambda_{N_{i}}\sigma_{R_{i}}^{2} \sin(\delta_{2} - 2\theta_{S}) \\ &- \lambda_{N_{i}}\sigma_{R_{j}}^{2} \sin(\delta_{2} - \theta_{R_{j}} + \theta_{L_{i}}) \Big) , \\ \frac{\partial V}{\partial \theta_{S}} &= 2\sigma_{S} \Big(\lambda_{H}\lambda_{S}v_{1}v_{2}\sigma_{S} \sin(\delta_{2} - 2\theta_{S}) \\ &- A_{H}v_{1}v_{2} \sin(\delta_{S} + \theta_{S}) + \xi^{3} \sin(\theta_{S}) \\ 1 &= e^{-1} \\ \end{bmatrix}$$

$$-\frac{1}{2}A_S\sigma_S^2\sin(3\theta_S) - \frac{1}{2}A_{N_i}\sigma_{R_i}^2\sin(\theta_S - 2\theta_{R_i}) -\frac{1}{2}\lambda_S\lambda_{N_i}\sigma_S\sigma_{R_i}^2\sin(2\theta_S + 2\theta_{R_i}) -\lambda_Hh_N^{ij}v_1\sigma_{R_j}\sigma_{L_i}\sin(\theta_S + \theta_{R_j} - \theta_{L_i}) +\lambda_{N_j}h_N^{ij}v_2\sigma_{R_j}\sigma_{L_i}\sin(\delta_2 - \theta_S + \theta_{R_j} + \theta_{L_i})),$$

$$\begin{split} \frac{\partial V}{\partial \theta_{R_i}} &= 2\sigma_{R_i} \left(\sigma_{R_i} \left(A_{N_i} \sigma_S \sin(\theta_S - 2\theta_{R_i}) \right. \\ &- \lambda_H v_1 v_2 \sin(\delta_2 + 2\theta_{R_i}) - \frac{1}{2} \lambda_S \sigma_S^2 \sin(2\theta_S + 2\theta_{R_i}) \right) \\ &- \sigma_S \sigma_{L_j} h_N^{ji} (\lambda_H v_1 \sin(\theta_S + \theta_{R_i} - \theta_{L_j}) \\ &+ \lambda_{N_i} v_2 \sin(\delta_2 - \theta_S + \theta_{R_i} + \theta_{L_j})) \\ &+ A_N^{ji} v_2 \sigma_{L_j} \sin(\delta_2 - \theta_{R_i} + \theta_{L_j}) \\ &+ h_N^{ji} h_N^{jk} v_2^2 \sigma_{L_k} \sin(\theta_{R_k} - \theta_{R_i}) \end{split}$$

$$+ \frac{1}{2} \lambda_{N_i} \lambda_{N_j} \sigma_{R_i} \sigma_{R_j}^2 \sin(2\theta_{R_j} - 2\theta_{R_i})$$

$$+ h_N^{ji} h_N^{kl} \sigma_{R_l} \sigma_{L_j} \sigma_{L_k} \sin(\theta_{R_l} - \theta_{R_i} + \theta_{L_k} - \theta_{L_j}) \bigg),$$

$$\frac{\partial V}{\partial \theta_{L_i}} = 2\sigma_{L_i} \left(\sigma_S \sigma_{R_j} h_N^{ij} (\lambda_H v_1 \sin(\theta_S + \theta_{R_j} - \theta_{L_i}) - \lambda_{N_j} v_2 \sin(\delta_2 - \theta_S + \theta_{R_j} + \theta_{L_i}) \right)$$

$$- A_N^{ij} v_2 \sigma_{R_j} \sin(\delta_2 - \theta_{R_j} + \theta_{L_i})$$

$$+ h_N^{ij} h_N^{kj} v_2^2 \sigma_{L_k} \sin(\theta_{L_k} - \theta_{L_i})$$

$$+ h_N^{ij} h_N^{kl} \sigma_{L_k} \sigma_{R_j} \sigma_{R_l} \sin(\theta_{L_k} - \theta_{L_i} + \theta_{R_l} - \theta_{R_j}) \bigg).$$

$$(5)$$

Here g_1 and g_2 are the U(1) and SU(2) gauge couplings, respectively, and $\tan \beta = v_2/v_1$. We use the seventeen minimization equations above to solve for the soft masses of the neutral scalar fields and a subset of A-parameters $(A_H, A_S, A_{N_i}, A_N^{i,3}).$

3 Higgs masses

Separating the real and imaginary parts ($\phi \equiv \phi_r + i\phi_i$) of the nine neutral scalar fields (two Higgs, one singlet and six sneutrinos) we get an 18×18 dimensional mass matrix for the scalars. The radiative corrections to the scalar masses are implemented via the one-loop effective scalar potential [67],

$$V_{1-\text{loop}} = \frac{-3}{32\pi^2} \left[\sum_{\tilde{a}=1}^4 m_{\tilde{a}}^4 \left(\log \frac{m_{\tilde{a}}^2}{\Lambda^2} - \frac{3}{2} \right) - \sum_{a=1}^4 m_a^4 \left(\log \frac{m_a^2}{\Lambda^2} - \frac{3}{2} \right) \right], \quad (6)$$

where $m_{\tilde{a}}^2$ are the field dependent eigenvalues of the 4×4 $\tilde{b}-\tilde{t}$ mass matrix and $m_t^2 = y_t^2 v_2^2$, $m_b^2 = y_b^2 v_1^2$. Λ is the renormalisation scale. The loop corrections lead to additional terms in both the minimisation conditions and the scalar mass matrix. In numerical calculations, we omit the *D*-term contributions and set for simplicity, $M_{Q_{33}} =$ $M_{U_{33}} = M_{\rm SUSY}$, with $M_{\rm SUSY} \sim \Lambda \sim 1$ TeV. The following experimental input is used:

$$v = 174 \text{ GeV}, \quad m_W = 80.42 \text{ GeV}, \quad m_t^{\text{Pole}} = 175 \text{ GeV}, \alpha_s = 0.102, \quad m_\tau = 1.777 \text{ GeV}, \quad \sin^2 \theta_w = 0.23124.$$
(7)

Here

$$m_t = \frac{m_t^{\text{Pole}}}{1 + \frac{4}{3\pi}\alpha_{\text{s}}} \,. \tag{8}$$

The rest of our free parameters are randomly sampled, with sampling ranges as follows (the couplings λ_i are constrained by perturbativity):

$$\begin{array}{ll} 0.1 < \lambda_{H,N_i} < 0.4 \,, & 0.2 < \lambda_S < 0.7 \,, & |h_N| < 10^{-7}, \\ 0.4 \, \text{TeV} < \xi < 1 \, \text{TeV} \,, & -\pi < \theta_\phi < \pi \,, & |\langle S \rangle| < 1 \, \text{TeV} \,, \\ |\langle \tilde{\nu}_L \rangle| < 100 \, \text{keV} \,, & |\langle \tilde{N}_i \rangle| < 1 \, \text{TeV} \,, & 2 < \tan \beta < 60 \,, \\ \end{array}$$

$$(9)$$

and the A-parameters not eliminated by (3) vary between $0 < A_N^{ij} < (1 \text{ TeV}) h_N^{ij}$. It should be noted that the VEVs of the right-handed sneutrinos are not constrained by any experimental bounds. In principle \tilde{N} could develop a VEV at a different scale than all the other scalars. However, if this VEV were to be complex, it would propagate CP violating phases to other (real valued) parameters of our model via their RG running equations and thus we restrict $\sigma_R \simeq M_{\rm SUSY}$.

In the limit $\lambda_{N_i} \to 0$ we recover lepton number conservation. CP and *R*-parity are still spontaneously violated as is lepton number, and thus in this limit the neutral scalar spectrum contains an additional Goldstone, as illustrated in Fig. 1. Using a model with only one right-handed neutrino (and consequently only one λ_N), the lightest scalar mass clearly tends to zero as the coupling $\lambda_N \to 0$.

For the full model we choose to set $\sigma_{R_{1,2}} = \theta_{R_{1,2}} = 0$ to further reduce the sampling space. This choice identically solves the vacuum conditions $\partial_{\theta_{R_1}} V = 0$ and $\partial_{\theta_{R_2}} V = 0$. The ZZh_i couplings (h_i denotes any neutral Higgs) are reduced compared to the SM, due to other than Higgs doublet components in the physical Higgs bosons. Thus the Higgs strahlung production is less frequent than in the SM and experimental bound on the lightest Higgs mass is reduced from the SM value of $m_H \gtrsim 114 \text{ GeV}$ [68,69], see, e.g., discussion in [70, 71]. Similarly the rate for the associated production of two Higgses through the couplings Zh_ih_i is reduced. In Fig. 2 we plot the mass of the lightest Higgs boson as a function of $\tan \beta$. In the figure we have applied the experimental limits from LEP [72, 73] on all neutral spin-0 particles to check that the masses are acceptable and we indicate the dominant component of the lightest Higgs. We have also studied the masses of the charged scalars at one-loop level and applied the experimental limits according to the main component of the charged scalar,

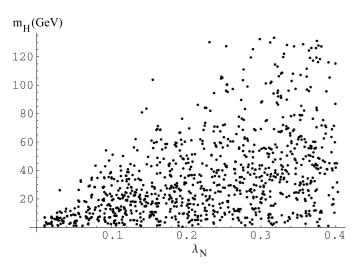


Fig. 1. Lightest scalar mass as a function of λ_N

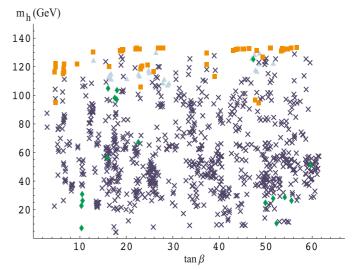


Fig. 2. The lightest physical scalar mass as a function of $\tan \beta$. The main component of the lightest scalar is $H_{1,2}^0$ (orange rectangle), S (green diamond), $\tilde{\nu}_L$ (light blue triangle), or \tilde{N} (dark blue cross)

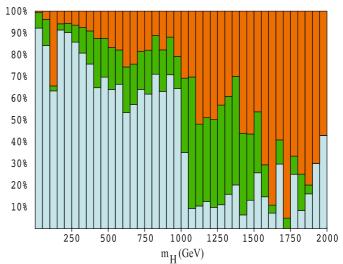


Fig. 3. Average composition of neutral Higgs particle as a function of the Higgs mass. *Lightest grey/blue* correspond to the sneutrinos, *medium grey/green* to singlet, and the *darkest grey/orange* to doublet Higgs

i.e. if the main component is stau, we have applied the experimental limit of 81.9 GeV, as appropriate for stau [74]. Since the charged scalar can be relatively light, it is interesting to consider the possibility of seeing it at Tevatron in the decay $t \to H^+b$. It appears that the light charged scalars are, however, mostly sleptons. Thus the coupling to quarks may be too weak to produce a significant branching ratio.

In Fig. 3 the composition of all the neutral Higgses is depicted as a function of their masses for one thousand parameter points satisfying the constraints mentioned above. It is seen that the light experimentally allowed Higgses tend to be mostly sneutrinos. In the region 100–150 GeV,

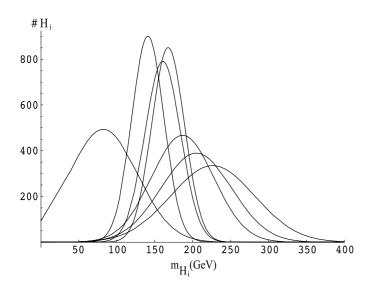


Fig. 4. Masses of the seven lightest neutral Higgses. The first peak corresponds to the lightest Higgs, second to second lightest etc.

a significant doublet Higgs component appears, as can also be seen from Fig. 2 showing that the lightest neutral Higgs is most often mostly doublet, if its mass is above 110 GeV. The heavier Higgses are mostly either singlets or doublets. In Fig. 4 a Gaussian is fit to the number vs. mass of the seven lightest neutral Higgses for one thousand parameter points. Curves for several of the lightest Higgses are strongly peaked, showing strong preference for particular mass values. The curves for heavier Higgses are much broader, showing much larger variation in their masses. Interestingly more than half a dozen are in the mass reach of the LHC. Unfortunately the doublet component in the light Higgses tends to be small, as seen in the Fig. 3 and their detection at the LHC may be challenging. Detailed study of the detection is beyond the present work.

4 Neutrino masses and mixing

Since the scalars VEVs appearing in the neutrino mass matrix include phases, it is expected that the neutrino sector of the model is CP violating.

of the model is CP violating. In a field basis of ν_{L_i} , N_i , \tilde{S} , \tilde{H}_1^0 , \tilde{H}_2^0 , \tilde{B} , \tilde{W} the neutral fermions form the following 11×11 mass matrix: The mass matrix (10) differs from the neutrino mass matrix in [39] by the phases for the VEVs. It is easy to see the structure of the usual seesaw mechanism, which produces small neutrino masses m_{ν} ,

$$M_{\chi^0} = \begin{pmatrix} 0 & m_D \\ m_D^{\rm T} & M_R \end{pmatrix}, \quad m_{\nu} = -m_D M_R^{-1} m_D^{\rm T}, \qquad (11)$$

where $m_D \ll M_R$. Similarly to [39], there are actually several sources for neutrino masses: the usual seesaw and the mixing of neutrinos with \tilde{h}_2^0 and gauginos through *R*-parity breaking.

It can be shown that the number of independent vectors in m_D is an upper bound of the number of non-zero neutrino masses, e.g., in models with exclusively the gaugino seesaw, there is at most one nonzero neutrino mass at tree-level because there is only one linearly independent vector, $\langle \tilde{\nu}_{L_i}^* \rangle$, in m_D . From (10) it is immediately apparent that we have four independent vectors in m_D : three in the Yukawa matrix $\mathbf{h}_N^{3\times3}$, and the vector of sneutrino vevs $\langle \tilde{\nu}_{L_i}^* \rangle$. If we were to include only one right-handed neutrino, there would be two linearly independent vectors, and as expected, we find that in such models there is one massless neutrino. In models with no right-handed neutrinos but bilinear R-parity violation, there are two independent vectors, $\mu_i \ (\equiv h_N^{i,j} \langle \tilde{N}_j^* \rangle$ in our model) and $\langle \tilde{\nu}_{L_i}^* \rangle$. The latter, however, can be rotated away using the accidental SU(4) symmetry of the $\{L_i, H_1\}$ fields, leaving only one independent vector and thus two massless neutrinos.

Inspecting the requirement that $m_D \ll M_R$ yields some qualitative features of the model. In particular, the left-handed sneutrino VEVs must be small and $h_N \langle \tilde{N}^* \rangle$ should be of the same order. Thus, although $\langle \tilde{N}^* \rangle$ is not bound by any other prior consideration, having $h_N \approx 10^{-7}$ results in an upper limit of a few TeV for the right-handed sneutrino VEVs.

We diagonalise M_{χ^0} numerically and use $M_1 \sim M_2 \sim 1$ TeV. Great care must be taken, as the elements of M_{χ^0} may vary over ten orders of magnitude, and the eigenvalues themselves over as much as twenty orders of magnitude. Our calculations are carried out using a forced minimal precision of fifty decimals. The errors due to the lack of precision in this case appear farther than eight places behind the decimal point for the neutrino masses. The diagonalising matrix \mathcal{N} , with $\mathcal{N}^* M_{\chi^0} \mathcal{N}^{-1} = \text{diag}(m_{\chi^0_i}, m_{\nu_j})$, has the following general form

$$M_{\chi^{0}} = \begin{pmatrix} \mathbf{0}_{3\times3} & \mathbf{h}_{N}^{3\times3} \langle H_{2}^{0} \rangle & \mathbf{0}_{3\times1} & \mathbf{0}_{3\times1} & h_{N}^{i,j} \langle \tilde{N}_{j}^{*} \rangle & -\frac{g_{1}}{\sqrt{2}} \langle \tilde{\nu}_{L_{i}}^{*} \rangle & \frac{g_{2}}{\sqrt{2}} \langle \tilde{\nu}_{L_{i}}^{*} \rangle \\ \mathbf{h}_{N}^{3\times3} \langle H_{2}^{0} \rangle & \mathbf{1}_{3\times3} \lambda_{N_{i}} \langle S \rangle & \lambda_{N_{i}} \langle \tilde{N}_{i}^{*} \rangle & \mathbf{0} & h_{N}^{j,i} \langle \tilde{\nu}_{L_{j}} \rangle & \mathbf{0} & \mathbf{0} \\ \mathbf{0}_{1\times3} & \lambda_{N_{i}} \langle \tilde{N}_{i}^{*} \rangle & \lambda_{S} \langle S \rangle & \lambda_{H} \langle H_{2}^{0} \rangle & \lambda_{H} \langle H_{1}^{0} \rangle & \mathbf{0} & \mathbf{0} \\ \mathbf{0}_{1\times3} & \mathbf{0} & \lambda_{H} \langle H_{2}^{0} \rangle & \mathbf{0} & \lambda_{H} \langle S \rangle & -\frac{g_{1}}{\sqrt{2}} \langle H_{1}^{0} \rangle & \frac{g_{2}}{\sqrt{2}} \langle H_{1}^{0} \rangle \\ h_{N}^{i,j} \langle \tilde{N}_{j}^{*} \rangle & h_{N}^{j,i} \langle \tilde{\nu}_{L_{j}} \rangle & \lambda_{H} \langle H_{1}^{0} \rangle & \lambda_{H} \langle S \rangle & \mathbf{0} & \frac{g_{1}}{\sqrt{2}} \langle H_{2}^{0*} \rangle & -\frac{g_{2}}{\sqrt{2}} \langle H_{2}^{0*} \rangle \\ -\frac{g_{1}}{\sqrt{2}} \langle \tilde{\nu}_{L_{i}}^{*} \rangle & \mathbf{0} & \mathbf{0} & -\frac{g_{1}}{\sqrt{2}} \langle H_{1}^{0} \rangle & \frac{g_{1}}{\sqrt{2}} \langle H_{2}^{0*} \rangle & M_{1} & \mathbf{0} \\ \frac{g_{2}}{\sqrt{2}} \langle \tilde{\nu}_{L_{i}}^{*} \rangle & \mathbf{0} & \mathbf{0} & \frac{g_{2}}{\sqrt{2}} \langle H_{1}^{0} \rangle & -\frac{g_{2}}{\sqrt{2}} \langle H_{2}^{0*} \rangle & \mathbf{0} & M_{2} \end{pmatrix} \right).$$
(10)

$$\mathcal{N} = \begin{pmatrix} \zeta & N_{\chi} \\ V_{\nu}^{\mathrm{T}} & \bar{\zeta}^{\mathrm{T}} \end{pmatrix}.$$
(12)

Here $\zeta, \bar{\zeta} \ll 1$ denote 8×3 matrices that can be determined perturbatively, see e.g. [75–77]. Our interest lies in the matrix V_{ν} , the neutrino mixing matrix. Using the canonical notation for the neutrino mixing matrix [75–77], $U = V_{\nu} \cdot$ diag $(1, e^{i\phi_1}, e^{i\phi_2})$ and $c/s_{ij} = \cos/\sin\theta_{ij}$,

 $V_{\nu} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$ (13)

we can extract the mixing angles as follows:

$$\sin \theta_{13} = \left| V_{\nu}^{13} \right|, \quad \tan \theta_{12} = \left| \frac{V_{\nu}^{12}}{V_{\nu}^{11}} \right|, \quad \tan \theta_{23} = \left| \frac{V_{\nu}^{23}}{V_{\nu}^{33}} \right|.$$
(14)

One can also extract the CP violating Dirac phase δ [78]:

$$|\delta| = \sin^{-1} \left(\left| \frac{8 \operatorname{Im} \left(V_{\nu}^{21} V_{\nu}^{*22} V_{\nu}^{12} V_{\nu}^{*11} \right)}{\cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}} \right| \right), \quad (15)$$

where the Jarlskog invariant J_{CP} [79] of the neutrino sector is given by:

$$J_{\rm CP} = \left| \operatorname{Im} \left(V_{\nu}^{21} V_{\nu}^{*22} V_{\nu}^{12} V_{\nu}^{*11} \right) \right| = \left| \operatorname{Im} \left(V_{\nu}^{31} V_{\nu}^{*33} V_{\nu}^{13} V_{\nu}^{*11} \right) \right| \\ = \left| \operatorname{Im} \left(V_{\nu}^{23} V_{\nu}^{*22} V_{\nu}^{32} V_{\nu}^{*33} \right) \right|.$$
(16)

In the quark sector, the J values in the SM are known to be $J \sim 10^{-5}$. For points satisfying all the constraints from the scalar sector, we then apply the following experimental constraints concerning the neutrino sector [80]:

$$\begin{aligned} \sin^2 2\theta_{23} &\geq 0.89, \quad \sin^2 \theta_{13} \leq 0.047, \quad \sin^2 \theta_{12} \simeq 0.23 - 0.37, \\ \Delta m_{\rm atm}^2 &\simeq 1.4 \times 10^{-3} - 3.3 \times 10^{-3} \, {\rm eV}^2, \\ \Delta m_{\rm sol}^2 &\simeq 7.3 \times 10^{-5} - 9.1 \times 10^{-5} \, {\rm eV}^2. \end{aligned}$$
(17)

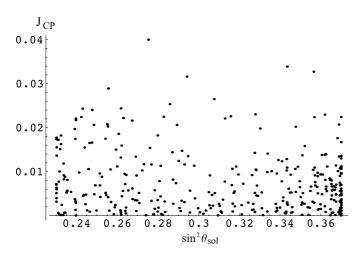


Fig. 5. Jarlskog parameter as a function of the solar angle

In Fig. 5 we show Jarlskog invariant in (16) for a sample of 350 points in the parameter space satisfying all the scalar and neutrino sector constraints. It is seen that $J_{\rm CP}$ is less than around 0.04. These values open the possibility of detecting the CP violation in the leptonic sector through neutrino oscillations, see [81, 82] and references therein.

5 Fermion electric dipole moments and CP violation in the kaon system: ϵ_K

Electric dipole moments represent a challenge for supersymmetric theories. It is known that the MSSM predicts too large EDMs by about three orders of magnitude for scalar fermion masses close to the current experimental bounds ($\mathcal{O}(100 \text{ GeV})$) and CP violating phases of $\mathcal{O}(1)$ [9–11]. There are at present three solutions to this problem. One is to assume that supersymmetric phases are not of order unity, but rather of $\mathcal{O}(10^{-2}-10^{-3})$ [9–11]. The second possibility is that the spectrum of the supersymmetric partners of quarks and leptons is heavy, i.e. of $\mathcal{O}(3 \text{ TeV})$ or more [83, 84], and out of reach of the LHC. The third possibility is that there are internal cancellations among the different components of the neutron EDM (the chargino and gluino contributions in particular) which can reduce the magnitude of the neutron EDM [85–88]. Analyses have demonstrated that these cancellations are very difficult to achieve [89–91]. Finally attempts to set the flavor diagonal CP violation parameters to zero, but to allow CP violation through off diagonal elements in the scalar fermion mass matrices lead to too large EDMs, and further constraints must be imposed [92–95]. All these solutions are in effect fine tuning, either for the scalar fermion masses, or for the phases, or for part of the parameter space.

The EDMs and ϵ_K in our model arise from the loop contributions shown in Figs. 6 and 7 and are straightforward to calculate. The definition of the EDM d_f for a spin- $\frac{1}{2}$ particle is

$$\mathcal{L}_I = -\frac{\mathrm{i}}{2} d_f \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi F^{\mu\nu}, \qquad (18)$$

and the general interaction Lagrangian between two fermions $(\bar{\Psi}, \Psi)$ and a scalar (χ) containing CP violation is

$$-\mathcal{L}_{\text{int}} = \sum_{ik} \bar{\Psi}_f \left(K_{ik} \frac{1-\gamma_5}{2} + L_{ik} \frac{1+\gamma_5}{2} \right) \Psi_i \chi_k + \text{h.c.} \quad (19)$$

This gives us the one loop EDM as

$$d_f = \sum_{ik} \frac{m_i}{(4\pi)^2 m_k^2} \operatorname{Im}(K_{ik} L_{ik}^*) \\ \times \left[Q_i A\left(\frac{m_i^2}{m_k^2}\right) + Q_k B\left(\frac{m_i^2}{m_k^2}\right) \right], \quad (20)$$

$$A(r) = \frac{1}{2(1-r)^2} \left(3 - r + \frac{2\log r}{1-r} \right), s \tag{21}$$

$$B(r) = \frac{1}{2(1-r)^2} \left(1 + r + \frac{2r\log r}{1-r} \right).$$
(22)

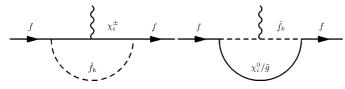


Fig. 6. The loop contributions to fermion EDMs

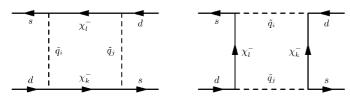


Fig. 7. The loop contributions to ϵ_K

Where Q_i , Q_k are the charges of the fermion and scalar respectively. Clearly we need $\operatorname{Im}(K_{ik}L_{ik}^*) \neq 0$ for there to be a nonzero contribution. There are three different contributions depending on which particles are running in the loop: chargino, neutralino or gluino. As mentioned above, small EDMs are achieved if these contributions cancel out. From (20) the other two common solutions are also easily understood, since increasing the squark mass m_k or suppressing $\operatorname{Im}(K_{ik}L_{ik}^*) \neq 0$ both yield a small d_f .

The CP-violating parameter $\epsilon_K = \text{Im}(\mathcal{M}_{K\bar{K}}/\Delta m_K)$ receives no contribution from standard model processes in our model and the only contribution to $\mathcal{M}_{K\bar{K}}$ comes from a chargino loop. Compared to the EDMs, the calculation of the kaon oscillation loop is more involved because there are non-perturbative hadronic states in the process. We use the vacuum insertion approximation (VIA) [24] whereby the matrix element $\langle K | (\bar{s}\Gamma d) | \bar{s}\Gamma d) | K \rangle$ appearing in the loop calculation is reduced to two nonzero contributions (\mathbf{V}_1 and \mathbf{V}_2) which can be measured from kaon decays.

Defining the quark-squark-chargino interaction as

$$-\mathcal{L}_{q\bar{q}\chi} = \bar{q}_i \left(V_{ijk}^L \gamma_L + V_{ijk}^R \gamma_R \right) \tilde{q}_j \chi_k + \text{h.c.}, \qquad (23)$$

and introducing the following notation:

$$\begin{split} W^{1}_{ijkl} &= V^{L*}_{sil}V^{L}_{djl}V^{L*}_{sjk}V^{L}_{dik} + (L \leftrightarrow R) \,, \\ W^{2}_{ijkl} &= V^{L*}_{sil}V^{L}_{djl}V^{R*}_{sjk}V^{R}_{dik} + (L \leftrightarrow R) \,, \\ W^{3}_{ijkl} &= V^{L*}_{sil}V^{R}_{djl}V^{L*}_{sjk}V^{R}_{dik} + (L \leftrightarrow R) \,, \\ W^{4}_{ijkl} &= V^{L*}_{sil}V^{R}_{djl}V^{R*}_{sjk}V^{L}_{dik} + (L \leftrightarrow R) \,, \end{split}$$

we can write the matrix element as

$$\mathcal{M}_{K\bar{K}} = \frac{i}{(2\pi)^4} \frac{1}{16\pi^2} \sum_{ijkl} \left\{ \frac{2}{m_i^2} \mathbf{I}_{ijkl}^1 \right. \\ \times \left[W_{ijkl}^1 \frac{2}{3} \mathbf{V}_2 + W_{ijkl}^2 \left(\frac{1}{3} \mathbf{V}_1 - \frac{1}{2} \mathbf{V}_2 \right) \right] \\ \left. + \frac{M_l M_k}{m_i}^4 \mathbf{I}_{ijkl}^2 \right. \\ \times \left[W_{ijkl}^3 \frac{5}{12} \mathbf{V}_1 + W_{ijkl}^4 \left(-\frac{1}{2} \mathbf{V}_1 + \frac{1}{12} \mathbf{V}_2 \right) \right] \right\}.$$
(24)

Here \mathbf{I}^i are lengthy expressions arising from the loop integrals depending on the masses of the particles in the loop. Denoting $R_{ab} \equiv (m_a^2 - m_b^2)/m_i^2$, they are

$$\begin{split} \mathbf{I}_{ijkl}^{1} &= \\ R_{lj}^{-1} \left\{ R_{jk}^{-1} \left[R_{ik}^{-1} \left(\frac{(1 - R_{ik})^{2}}{2} \left(\log(1 - R_{ik}) - \frac{1}{2} \right) + \frac{1}{4} \right) \right. \\ &- R_{ij}^{-1} \left(\frac{(1 - R_{ij})^{2}}{2} \left(\log(1 - R_{ij}) - \frac{1}{2} \right) + \frac{1}{4} \right) \right] \\ &- R_{lk}^{-1} \left[j \rightarrow l \right] \right\}, \end{split}$$
(25)
$$\mathbf{I}_{ijkl}^{2} &= R_{lj}^{-1} \left\{ R_{jk}^{-1} \left[R_{ik}^{-1} (1 - R_{ik}) \log(1 - R_{ik}) \right. \\ &- R_{ij}^{-1} (1 - R_{ij}) \log(1 - R_{ij}) \right] \\ &- R_{lk}^{-1} \left[R_{ik}^{-1} (1 - R_{ik}) \log(1 - R_{ik}) \right. \\ &- R_{il}^{-1} (1 - R_{il}) \log(1 - R_{ik}) \\ &- R_{il}^{-1} (1 - R_{il}) \log(1 - R_{il}) \right] \right\}. \end{split}$$
(26)

It is easily seen how these expressions simplify when using the mass insertion approximation (i.e. i = j). Since the model provides us with a full squark spectrum, we prefer to use the full expressions in (24). The VIA coefficients are given as follows [24]:

$$\mathbf{V}_1 = \frac{f_K^2 m_K^4}{2m_K (m_s + m_d)^2}, \quad \mathbf{V}_2 = \frac{f_K^2 m_K^2}{2m_K}, \quad f_K \simeq 160 \text{ MeV}.$$
(27)

Using these results, our method to search for viable points in the parameter space has been to find first points which satisfy all the Higgs mass and neutrino sector constraints. From these points we begin a random walk in parameter space, slightly varying the parameter values for each step, checking the EDMs and discarding such steps as do not bring us closer to the experimentally acceptable values. There is a set of parameters (A_u, M_3, M_U^{11}) which enters exclusively in the calculation of the EDM. Suppression of the EDM can be achieved by increasing these mass parameters or, alternatively, one can find values for which cancellation between the different loop contributions occur. In general, varying only these parameters can lead to undesirably large mass parameter values (> $\mathcal{O}(3 \text{ TeV})$). In such cases we vary all the parameters relevant to the EDM. Since the same parameters affect other sectors of our model as well, we have to check, at each random step, the various experimental constraints. In Fig. 8 we show the effect of changing λ_{N_3} on the neutron EDM. The curve shows two values for λ_{N_3} where cancellations between the contributions to the EDM occur. It is seen that only one of the two dips in the curve satisfies all the required constraints. After this we try, again by randomly varying parameters, to find experimentally acceptable ϵ_K . This process is easier than for the EDMs, since the set of parameters $(A_{\rm c}, M_Q^{22}, M_U^{22})$, which only affects the value of ϵ_K , is sufficient for reaching acceptable ϵ_K values. In Fig. 9 the behaviour of ϵ_K , for the same parameter point as in Fig. 8, is shown as a function of one of the trilinear couplings, $A_{\rm c}$. A clear peak where we find the experimentally allowed value can be seen. For the range shown, all the scalar

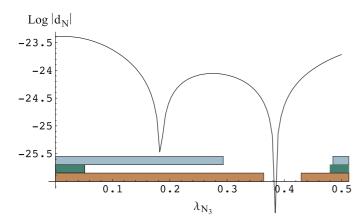


Fig. 8. λ_{N_3} effect on neutron EDM. The *bars* show excluded ranges due to problems with vacuum stability (*top*), charged Higgs mass limits (*middle*), and neutrino sector constraints (*bottom*)

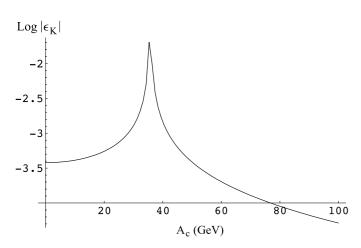


Fig. 9. ϵ_K as a function of the trilinear coupling A_c

and neutrino sector constraints, as well as EDMs, remain viable.

We find that satisfying EDMs and kaon sector CP violation is possible without contstraining the phases, though quite restrictive, in the model. The reason why satisfactory values are found is due to the structure of the parameter space. There are variables, which affect EDMs or ϵ_K , but which do not affect the other observables.

6 Conclusions and discussion

Violation of CP symmetry is well established, while neutrino masses make violation of R-parity attractive. We have considered here a model in which both R-parity and CP-symmetry break spontaneously. Our model contains in addition to the MSSM fields only the three righthanded neutrinos N_i and a singlet field S, which are both needed to spontaneously break CP and R-parity. We have shown that experimentally viable neutrino and Higgs boson masses can be obtained, and that CP is violated in the neutrino sector. We explored the parameter space requiring that EDM bounds and experimental results on the kaon system (especially ϵ_K) are satisfied. In [54, 55] solutions for models satisfying both constraints in an *R*-parity conserving model were discussed. In our case, there are more parameters in relevant sectors, like the chargino mass matrix, which make the bounds mentioned easier to fulfill.

While this paper is dedicated to the presentation of the model, and constraints on its parameter space, two immediate consequences arise for low energy phenomenology. In the leptonic sector, we predict a measurable CP violation, with the Jarlskog invariant below 0.04. In the scalar sector, we predict reduced ZZh_i couplings (with h_i any neutral Higgs boson) compared to the SM, and a Higgs strahlung production less frequent than in the SM. As well, the model favors a lightest Higgs mass less than its SM value, $m_H < 114$ GeV. With our particle content, it is kinematically possible to produce several neutral Higgses within the LHC mass reach.

In the present paper we have not studied the B-meson sector in detail, and we expect that modifications for the quark sector are needed before we can agree with the experimental results. Such modifications are achieved e.g. by using a model where the Higgs sector leads to a complex CKM matrix [57, 58] or by adding vector quarks [65, 66]. However, the parameters involved in the calculations of the neutrino sector and calculations involving quarks are largely disconnected, as we have seen from our kaon sector results. Thus the results of the present model concerning the Higgs and neutrino sectors are not expected to change qualitatively. The work along these directions is in progress [96].

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